Example Find the general solution of

$$70x+112y=168$$

Solution Here

$$70x+112y=168 \quad \text{——— (1)}$$

firstly we shall find the gcd of 70 and 112

$$112 = 70(1)+42$$
$$70 = 42(1)+28$$
$$42 = 28(1)+14$$
$$28 = 14(2)+0 \Rightarrow (70,112)=14$$

$\Rightarrow 14/168$ so equation (1) has solution.

Dividing (1) by 14, we get

$$5x+8y=12$$

We can easily see that $x=-4$ and $y=4$ satisfy the above equation. Its general solution is

$$x_1 = x_0 - \frac{112}{14}t = -4-8t$$

and $y_1 = y_0 - \frac{70}{14}t = 4+5t$

EX Solve the diophantine equation

$$525x+231y=42 \quad \text{——— (1)}$$

Solution: We can easily find that

$$(525,231)=21$$

therefore dividing (1) by 21 we get

$$25x+11y=2$$

Again $(25,11)=1$ by the euclid's algorithms; we get

$$25(4)+11(-9)=1$$

Hence $x=2\cdot4=8$, $y=2(-9)=-18$ is a solⁿ of equation

Therefore general solution is $x=8+11t$, $y=-18-25t$

Theorem. If $ax+by=c,\ (a,b)=1$ ——— (1)

b is numerically smaller of the two coefficients a and b and a_1 and c_1 are the minimal remainders of a and c respectively with respect to $|b|$. Then (1) can be written in the form

$$a_1x+|b|x_1=c_1$$

where $|a_1|\le\frac{|b|}{2}$ and $|c_1|\le\frac{|b|}{2}$

Proof:→ given that a_1 and c_1 are minimal remainders of a and c with respect to $|b|$,

we have

$$a=|b|q_1+a_1\ ,\quad 0<a_1\le\frac{|b|}{2}$$

$$c=|b|q_2+c_1\ ,\quad 0<|c_1|\le\frac{|b|}{2}$$

Therefore (1) reduces to

$$(|b|q_1+a_1)x+by=|b|q_2+c_1$$

or $a_1x+|b|\left(q_1x+\frac{b}{|b|}y-q_2\right)=c_1$

putting $x_1=q_1x+\frac{b}{|b|}y-q_2$ the above equation reduces to

$$a_1x+|b|x_1=c_1$$

Example Find the possible solution of

$$11x+5y=79 \quad\text{——— (1)}$$

Solution clearly $(11,5)=1$

Now $11=5.2+1$

$79=5.16-1$

Then (1) can be written as

$$(5.2+1)x+5y=5\times16-1$$

$\Rightarrow 5[2x+y-16]+x=-1$

$\Rightarrow 5u+x=-1$ where $u=2x+y-16$

putting $u=0$, we get $x=-1$ then from (1) we get $y=18$

$\Rightarrow x=-1, y=18$ is one solution.

The general solution is given by

$$x=-1+5t, \quad y=18-11t$$

Since we require the solution to be positive, therefore, we have to find the value of t for which x and y are positive.

putting $t=1$ we get

$$x=-1+5\times 1=4$$

$$y=18-11\times 1=7$$

Further, for $t=2$ and >2, y will be positive

Hence, the only positive solution is $x=4$ and $y=7$